

Exam 2 – Voltage and Moving Charges

March 7, 2013

This is a closed book examination but during the exam you may refer to a 3"x5" note card with words of wisdom you have written on it. There is extra scratch paper available. Your explanation is worth $\frac{3}{4}$ of the points. Explain your answers!

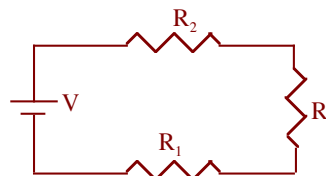
A general reminder about problem solving:

- Show all your work.
- Really; Show All Work!
- Focus
 - Draw a picture of the problem
 - What is the question? What do you want to know?
 - List known and unknown quantities
 - List assumptions
- Physics
 - Determine approach – What physics principles will you use?
 - Pick a coordinate system
 - Simplify picture to a schematic (if needed)
- Plan
 - Divide problem into sub-problems
 - Modify schematic and coordinate system (if needed)
 - Write general equations
- Execute
 - Write equations with variables
 - Do you have sufficient equations to determine your unknowns?
 - Simplify and solve
- Evaluate
 - Check units
 - Why is answer reasonable?
 - Check limiting cases!

1. [4 PTS] An object with negative charge, $q = -7 \text{ uC}$, is moving with a constant speed along the x-axis, $\vec{v} = \langle 8 \times 10^4, 0, 0 \rangle \text{ m/s}$. When the object is at $\vec{x} = \langle 5, 0, 0 \rangle \text{ m}$, where is the magnetic field produced by this moving charge zero?
- a) The magnetic field is zero everywhere.
 - b) $\vec{x} = \langle 5, 5, 0 \rangle \text{ m}$
 - c) $\vec{x} = \langle 5, 0, 5 \rangle \text{ m}$
 - d) $\vec{x} = \langle 10, 0, 0 \rangle \text{ m}$**
 - e) The magnetic field is not zero anywhere.

According to the Biot-Savart law $\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$ so the magnetic field is zero if $q=0$, $v=0$ or if the velocity and observation vector are parallel.

2. [4 PTS] You connect three light bulbs (resistors) to a battery as shown in the diagram to the right. If the light bulb labeled R_2 is dimmest (i.e. uses least power), what must be true?
- a) R_2 has the largest resistance.
 - b) R_2 has the same resistance as the other light bulbs, it just has the current (which is really electrons traveling the opposite direction) pass through it last.
 - c) R_2 has the smallest current.**
 - d) R_2 has the smallest resistance.
 - e) R_2 has the largest voltage drop.



The current is the same in series and $P=IV=I^2R$ so the smallest resistance has the smallest power.

The next two problems concern a long wire positioned along the x-axis that has a current of $\vec{I} = \langle 1.2, 0, 0 \rangle A$ flowing through it. The wire's center is at the origin, $\vec{x} = \langle 0, 0, 0 \rangle m$.

3. [4 PTS] What is the direction of the magnetic field directly above the wire at

$\vec{x} = \langle 0, 2, 0 \rangle mm$?

- a) $\vec{B} \propto \langle 0, 1, 0 \rangle T$
- b) $\vec{B} \propto \langle 0, -1, 0 \rangle T$
- c) $\vec{B} \propto \langle 1, 0, 0 \rangle T$
- d) $\vec{B} \propto \langle -1, 0, 0 \rangle T$
- e) $\vec{B} \propto \langle 0, 0, 1 \rangle T$
- f) $\vec{B} \propto \langle 0, 0, -1 \rangle T$

Current is traveling in the $+\hat{x}$ direction and the observation point is in the $+\hat{y}$ direction use the right hand rule to find magnetic field in the $+\hat{z}$

4. [4 PTS] The current in the wire is doubled and you move twice as far away. What happens to the magnitude of the magnetic field your new observation point?

- a) Increases by a factor of 4
- b) Increases by a factor of 2
- c) Does not change
- d) Decreases by a factor of 2
- e) Decreases by a factor of 4

$B \propto \frac{I}{r}$ so if both change (increase by 2) then there is no change to the magnitude of the magnetic field.

5. [4 PTS] A neutral hollow metal sphere is placed between two large charged plates. The plates are 15 cm apart. The voltage difference between the plates is kept at a high voltage of 150kV. What is the voltage inside the hollow metal sphere?

- a) The voltage is zero.
- b) The voltage increases toward the center.
- c) The voltage is constant inside the sphere.
- d) The voltage decreases toward the center.
- e) There is no voltage inside the sphere.

$\vec{E} = -\frac{d}{dr}V\hat{r}$ or $V = -\int \vec{E} \cdot d\hat{r}$ and we know the electric field inside a conductor in static equilibrium is zero so the voltage must be constant i.e. the slope of a constant is zero.

6. [4 PTS] You connect three light bulbs in parallel to a 9 V battery. One of the light bulbs is noticeably brighter (uses more power) than the other two light bulbs. What happens when you disconnect this bright bulb? Assume the battery is ideal – it can supply any amount of current.

- a) The other two light bulbs increase in brightness.
- b) The bulb closest to the battery increases in brightness.
- c) Nothing happens to the brightness of the other two bulbs.
- d) The bulb furthest from the battery decreases in brightness.
- e) The other two light bulbs decrease in brightness.

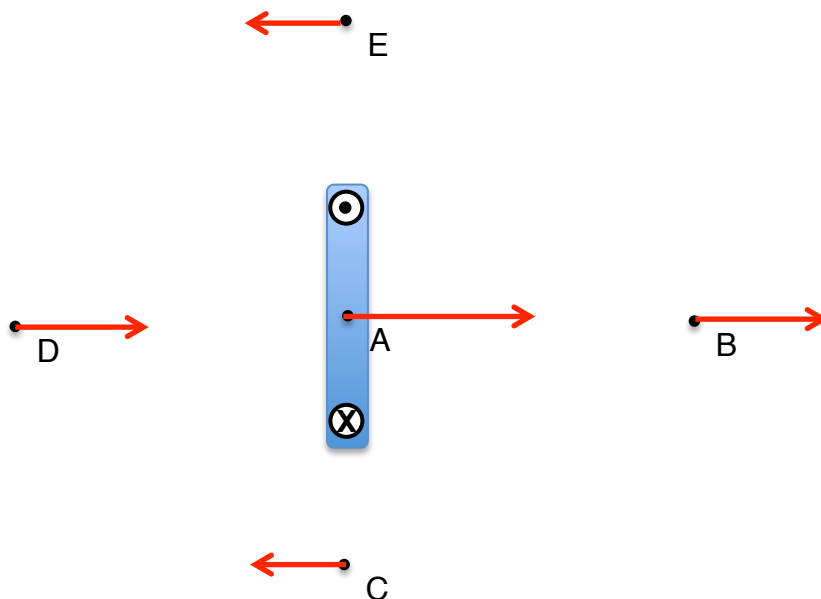
The voltage across all the circuit elements connected in parallel is the same. Since $P = IV = \frac{V^2}{R}$ we see that the brightness of the other bulbs is not affected.

7. [4 PTS] A resistor is placed in series with a 1.5 volt battery. What is the resistance if you measure a current of 0.25 A in this circuit?

- a) 0.167 Ω
 b) 0.375 Ω
 c) 2.67 Ω
 d) 6.00 Ω
 e) 24.0 Ω

$$\text{For an ohmic device } V = IR \text{ so } R = \frac{V}{I} = \frac{1.5V}{0.25A} = 6\Omega$$

8. [4 PTS] The diagram to the right is the side of a wire loop with a current flowing in it. The current flows into the bottom of the loop and out of the top of the loop. Indicate the direction of the magnetic field at the 5 locations indicated.



Use the Biot-Savart law in current form and the right-hand-rule to find the direction of the magnetic field.

$$\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{l} \times \hat{r}}{r^2}$$

9. [4 PTS] A negative point charge is located at the origin. You measure the voltage 2 cm from the charge. What is the voltage 4 cm from the charge?

- a) The voltage increases by a factor of 4
 b) The voltage increases by a factor of 2
 c) The voltage does not change
 d) The voltage decreases by a factor of 2
 e) The voltage decreases by a factor of 4

The voltage depends on the distance as well as the sign of the charge $V = \frac{kq}{r}$. Hence the voltage will change by a factor of 2 and become less negative so it increases in value.

10. [4 PTS] Two charges form a dipole centered at the origin. Both charges have the same magnitude $|q| = 3.21 \times 10^{-19} C$ and have a center-to-center separation of 8.75 nm. What is the electric potential at the origin (midway between the charges)?

- a) 0.66 V
 b) 0.33 V
 c) 0 V
 d) -0.33 V
 e) -0.66 V

The net voltage is the sum of the voltage from each charge $V = \frac{kq}{r}$ so since the charges are opposite in sign and equal in distance the net voltage is zero.

The next two problems can be done using problem solving sheets or on additional paper.

11. [12 PTS] An electron is released from a heated filament (basically a light bulb). The filament is at a voltage of 120 V. There is a large plate 2 cm from the filament. You would like the velocity of the electron to be 3×10^6 m/s (1% the speed of light!) when it reaches the plate. Determine the necessary voltage on the plate.
 BONUS [10%]: If you apply relativistic methods to this problem.

Use conservation of energy to find $\frac{1}{2}mv^2 = q\Delta V$ Solve to find the required voltage change $\Delta V = \frac{mv^2}{2q} = \frac{9.109 \times 10^{-31} \text{ kg } (3 \times 10^6 \frac{\text{m}}{\text{s}})^2}{2(1.602 \times 10^{-19} \text{ C})} = 25.6 \text{ V}$ so the plate must have a greater voltage (attractive for an electron) of $V=145.6 \text{ V}$

Relativistic methods need to be employed if γ is much greater than one. Check using $\beta = \frac{v}{c} = 0.01$ so $\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} = 1.000050$ Hence, this is not a relativistic problem.

12. [12 PT] You are given a 15W light bulb and a 150W light bulb. The power ratings for these light bulbs refer to a standard configuration where the bulbs are hooked in parallel to a 120 V power supply. For some reason you decide to connect these light bulbs in series to the 120 V power supply. Determine the power ratings for the bulbs in this new configuration.

Initially the light bulbs are in parallel so use $P = IV = \frac{V^2}{R}$ to determine the resistance of each light bulb $R_{15W}=960\Omega$ and $R_{150W}=96 \Omega$ which when the bulbs are connected in series they each have the same current flowing through them. In series the resistances add so $R_{tot} = R_{15W} + R_{150W} = 1056\Omega$ and since $V = IR_{tot}$ the total current in the new circuit is $I = 0.114 \text{ A}$. Use $P = IV = I^2R$ to find the power each bulb now uses in series. The 15W light bulb now uses 12.4W and the 150W light bulb now uses 1.24W. As expected the lower resistance element uses the least power in series and the most power in parallel.

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{\text{N}}{\text{A}^2} \quad \frac{1}{4\pi\epsilon_0} = k = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

mass of electron $m_e = 9.109 \times 10^{-31} \text{ kg}$
 charge of electron $q_e = 1.602 \times 10^{-19} \text{ C}$

$$V_{\text{sphere}} = \frac{4\pi r^3}{3} \quad \text{and} \quad A_{\text{sphere}} = 4\pi r^2$$

mass of proton $m_p = 1.673 \times 10^{-27} \text{ kg}$